RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)

THIRD YEAR
B.A./B.SC. FIFTH SEMESTER (July – December), 2012
Mid-Semester Examination, September 2012

Date : 10/09/2012 — **MATHEMATICS (Honours)**
Time : 2 pm – 4 pm — Paper : V — Full Marks : 50

[Use Separate Answer Scripts for each group]

Group – A

Answer **any five** questions : [5×5]

1. Let G be a group and H be a subgroup of G. Define $N(H) = \{g \in G : gHg^{-1} = H\}$. Prove that N(H) is a subgroup of G. Also show that if [G:H] = 2 then H is normal in G. [3+2]

2. Prove that the centre Z(G) of a group G is a normal subgroup of G. Prove also that the symmetric group S_3 has a trivial centre. [2+3]

3. a) Let G be a non-commutative group of order 2p, p is prime. Without using Cauchy's theorem prove that G is not simple.
   b) Le G be a group and $a \in Z(G)$. Prove that <a> is normal in G. [3+2]

4. a) Let G = (R, +), G′ = (T, ·) where $T = \{z \in C : |z| = 1\}$. Prove that $G/Z \cong G'$.
   b) Prove that the groups (R, +) and (R*, ·) are not isomorphic where R* = R –{o}. [3+2]

5. Define inner automorphism of a group G. Prove that $G/Z(G) \cong \mathrm{Inn}(G)$. [1+4]

6. a) If G = (Z, +), find Aut (G).
   b) Prove that there does not exist an onto homomorphism from (Z_6, +) to (Z_4, +). [3+2]

7. Let H, K be normal subgroups of a group G. If G is an internal direct product of H and K then prove that $G \cong H \times K$ and $G/H \cong K$. [5]

8. a) Prove that Z×Z is not cyclic.
   b) Show that (Z, +) is not an internal direct product of two nontrivial subgroups. [3+2]

Group – B

Answer **any three** : [3×5]

9. Deduce Newtons forward interpolation polynomial. What is the importance of introducing u? [4+1]

10. What is numerical differentiation? Obtain numerical differentiation for Lagranges interpolation polynomial at a point which is not a node. [2+3]

11. Obtain Newton Cotes formula for numerical integration. Deduce Simpson's $1/3$ rule from it. [3+2]

12. Discuss the method of bisection for numerical solution of the equation f(x) = 0. Why it is a sure convergent method? [4+1]

13. Formulate Newton Raphson method for finding numerical solution of f(x) = 0. Why it is called a method of tangents. [4+1]

Answer **<u>any two</u>** : [2×5]

14. a) Prove by Mathematical Induction that $3^{2n} - 8n - 1$ is divisible by 64 for all $n \in N$.

    b) If a is prime to b, prove that a + b is prime to ab. [3+2]

15. a) Prove that any positive integer is either 1 or a prime or can be expressed as product of primes.

    b) If p is prime and p | ab prove that p | a or p | b. [3+2]

16. a) State and prove Euclid's theorem.

    b) If p is prime > 3 prove that $24 \mid p^2 - 1$. [3+2]